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EFFECT OF ROTATION ON THE ELECTROGASDYNAMIC CHARACTERISTICS OF FLOW IN A CYLINDRICAL CHANNEL

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The study of flows of unipolarly charged gas streams is of definite theoretical and practical interest, particularly in designing electrogasdynamic energy converters [1, 2].

Discussed below is the problem of the influence of prerotation on the electrogasdynamic characteristics of flow in an ideally conducting round tube with grounded walls. In addition to applications to the theory of propagation of EHD streams, such a problem is also of independent importance, since it makes it possible to explain certain characteristics of the motion of charged fluids in pipes [3, 4].

The description of the flow field uses two-dimensional axisymmetric Navier–Stokes equations, written in terms of the variables, stream function versus vorticity. It is assumed that the energy of the gas flow is much greater than that of the electric field, so that the flow is not sensitive to electric processes, i.e., the distribution of the flow parameters can be found before the "electric" characteristics. Thus, the flow equations in a dimensionless field will be written

$$\frac{\mathrm{Re}}{4}\xi^2\left\{\frac{\partial}{\partial x}\left[\frac{\omega}{\xi}\frac{\partial\psi}{\partial\xi}\right]-\frac{\partial}{\partial\xi}\left[\frac{\omega}{\xi}\frac{\partial\psi}{\partial x}\right]\right\}-\left\{\frac{\partial}{\partial x}\left[\xi^3\frac{\partial(\omega/\xi)}{\partial x}\right]+\frac{\partial}{\partial\xi}\left[\xi^3\frac{\partial(\omega/\xi)}{\partial\xi}\right]\right\}-\mathrm{Re}\,\xi\frac{\partial V_\theta^2}{\partial x}=0,$$

$$\frac{\partial}{\partial x}\left[\frac{1}{\xi}\frac{\partial\psi}{\partial x}\right]+\left[\frac{1}{\xi}\frac{\partial\psi}{\partial\xi}\right]+\omega=0, \qquad (1)$$

$$\frac{\mathrm{Re}}{4}\left\{\frac{\partial}{\partial x}\left[V_\theta\xi\frac{\partial\psi}{\partial\xi}\right]-\frac{\partial}{\partial\xi}\left[V_\theta\xi\frac{\partial\psi}{\partial x}\right]\right\}-\left\{\frac{\partial}{\partial x}\left[\xi\frac{\partial V_\theta\xi}{\partial x}\right]+\frac{\partial}{\partial\xi}\left[\xi\frac{\partial V_\theta\xi}{\partial\xi}\right]\right\}+2\frac{\partial V_\theta\xi}{\partial\xi}=0.$$

In specifying the boundary conditions, it was assumed that at the entrance to the tube, the stream has a velocity distribution consistent with the law of rotation of a solid:

$$\psi=\xi^2,\quad \omega=0,\quad V_\theta=\sigma\xi \quad \text{for} \quad x=0.$$

Soft boundary conditions simulating the free outflow of a liquid were set at the exit:

$$\frac{\partial^2\psi}{\partial x^2}=0,\quad \frac{\partial^2\omega}{\partial x^2}=0,\quad \frac{\partial^2 V_\theta}{\partial x^2}=0 \quad \text{for} \quad x=100.$$

The following symmetry conditions hold on the tube axis:

$$\psi=0,\quad V_\theta=0 \quad \text{for} \quad \xi=0.$$

At the tube wall, the constancy of the flow rate was monitored and the adhesion conditions were simulated:

$$\psi=1,\quad V_\theta=0 \quad \text{for} \quad \xi=1.$$

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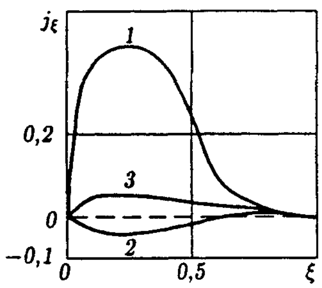

Fig. 1

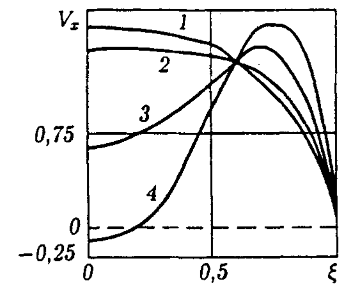

Fig. 2

The vorticity on the axis and wall of the tube was found in accordance with the method of [5]. The axial and radial velocity components were determined by numerical differentiation of the stream function:

$$V_x = \frac{1}{2\xi}\frac{\partial\psi}{\partial\xi}, \qquad V_\xi = -\frac{1}{2\xi}\frac{\partial\psi}{\partial x}.$$

Here x and ξ are the dimensionless longitudinal and axial coordinates referred to the tube radius R; ψ is the stream function; $\omega/\xi = 2\,\xi^{-1}(\partial V_\xi/\partial x - \partial V_x/\partial\xi)$ is the dimensionless tangential component of the vorticity; V_θ is the tangential velocity; $\sigma = \Omega R/U$ is a parameter characterizing the prerotation intensity at the entrance to the tube, and $Re = 2UR/\nu$ is the Reynolds number, constructed in terms of the mean flow-rate velocity $U = \int_0^1 V_x \xi d\xi$.

The modelling of the "electric" part of the problem was carried out under the following assumptions. The electric charges formed in the range $x < 0$, $\xi < \xi_i$ are introduced by the current into the tube. The size of the charge entrance cross section was $\xi_i < 1$. This cross section was characterized by a constant charge density q_i. Note that during the motion of charged particles in streams with recirculations, formation of regions where the longitudinal and transverse gradient q are of the same order is possible, and therefore, in the charge transfer equation, diffusion in the transverse direction will be taken into account. The dimensionless equations describing the distribution of potential Φ and charge transfer q take the form

$$\begin{gathered}\frac{\partial}{\partial x}\left[\xi\frac{\partial\Phi}{\partial x}\right] + \left[\xi\frac{\partial\Phi}{\partial\xi}\right] + q\xi = 0,\\ \frac{\text{Re Sc}}{4}\left\{\frac{\partial}{\partial x}\left[q\left(\frac{\partial\psi}{\partial\xi} - \frac{\partial\Phi}{\partial x}\right)\right] - \frac{\partial}{\partial\xi}\left[q\left(\frac{\partial\psi}{\partial x} + \frac{\partial\Phi}{\partial\xi}\right)\right]\right\} - \\ - \left\{\frac{\partial}{\partial x}\left[\xi\frac{\partial q}{\partial x}\right] + \frac{\partial}{\partial\xi}\left[\xi\frac{\partial(\omega/\xi)}{\partial\xi}\right]\right\} = 0;\end{gathered} \tag{2}$$

the boundary conditions for integrating this system of equations are

$$\begin{aligned} &x = 0: && q = q_i\ (\xi \le \xi_i), && q = 0\ (\xi > \xi_i),\\ &x = 100: && \frac{\partial^2 q}{\partial x^2} = 0, && \frac{\partial^2\Phi}{\partial x^2} = 0,\\ &\xi = 0: && \frac{\partial q}{\partial\xi} = 0, && \frac{\partial\Phi}{\partial\xi} = 0,\\ &\xi = 1: && q = 0, && \Phi = 0. \end{aligned}$$

Here the variables Φ, q are referred to the characteristic quantities UR/b and $\varepsilon U/(4\pi Rb)$; b is the charge mobility; Sc is Schmidt's number, taken as unity in the calculations.

A fairly simple and versatile method [5] was used to solve the system of equations (1), (2). The problem was solved with a 31×15 grid not uniform along the two coordinates and having concentration points at the entrance and wall of the tube. The finite-difference analogue of the system of differential equations was a system of nonlinear algebraic equations which was solved numerically by use of the Gauss–Seidel method. Convergence of the iterations at $\sigma \ge 3$ was ensured by using lower relaxation for the vorticity and velocity circulation $V_\theta\xi$. The convergence criterion of the iterations was the fulfillment of the inequality $|\mathbf{Y}^{(N)} - \mathbf{Y}^{(N-1)}| < 10^{-3}\,|\mathbf{Y}^{(N)}|$, where N is the iteration number, $\mathbf{Y} = (\psi, \omega/\xi, V_\theta\xi, q, \Phi)$.

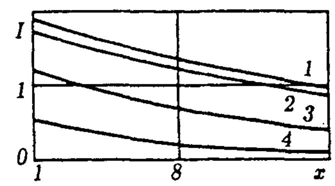

Fig. 3

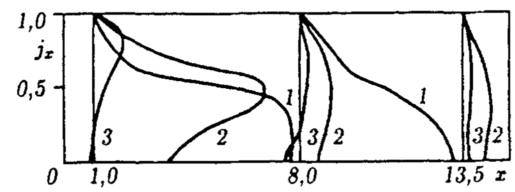

Fig. 4

Having thus completed the mathematical description of the problem, we now turn to the analysis of the results. The charged particles enter the channel, and in addition to moving in the longitudinal direction, begin to move in the transverse direction as well. The transverse motion of the particles is chiefly determined by two factors: transport of charged particles by the gas stream and motion of the charges with respect to the gas (when $b \neq 0$) under action of the electric field. Figure 1 shows the distribution of the transverse component of electric current

$$j_\xi = q(V_\xi - \frac{\partial \Phi}{\partial \xi}) - \mathrm{Re}^{-1}\frac{\partial q}{\partial \xi}$$

in the channel ($x = 0.7$). Curves 1, 2 correspond to a straight-through flow $\sigma = 0$ with $q_i = 1$ and $q = 10$, and curve 3 corresponds to a swirling flow $\sigma = 3$ with $q_i = 3$. It is evident that in the case of low mobility of the charge (small q_i), the main factor determining the charge transfer is convection. The outflow of gas from the wall region to the axial region as the dynamic boundary layer is being formed is what accounts for the negative j_ξ values in the region near the axis. As the charge mobility increases, a major role is played by the displacement of the charges under action of the electric field. The radial velocity of the charge, caused by radial component of the electric field $\partial\Phi/\partial\xi$, appreciably surpasses the radial velocity of convective flow. As can be seen from the figure, charged particles execute motion against the flow to the channel axis (curve 2).

Let us now turn to an analysis of the influence of rotation on the electrogasdynamic characteristics of the flow. We first note the basic characteristics of the velocity fields. At small σ (up to $\sigma = 1$), the influence of rotation on the velocity distribution is insignificant. When $\sigma > 2$, under action of the centrifugal forces arising in the flow, a region of reduced pressure is formed in the zone near the axis. As a result, formation of a V_x dip takes place in the vicinity of the axis. Such a region of reduced injection velocity is formed near the entrance to the tube, where, because of the interaction with the wall, boundary layers have already developed, but the tangential component of the velocity is still large. As the rotation increases further ($\sigma > 6$), the pressure gradients increase to such a degree that in the region near the axis, a reverse-flow region is formed whose size and shape as well as the flow rate therein are determined by the parameters Re and σ.

The deceleration of the stream in the region near the axis (and even its reversal at large σ) results in an increase of V_x at the periphery. The velocity diagram of V_x has a maximum at some $\xi_m \neq 0$. As σ increases, an increase in the maximum value of V_x and a displacement of ξ_m to the wall are observed. Downstream, friction forces cause the rotation to degenerate, and a Poiseuille velocity profile is formed. Figure 2 shows the distribution of axial velocity component in the section $x = 3$ for $\mathrm{Re} = 160$. Curve 1 represents a straight-through flow with $\sigma = 0$, and 2-4 represents swirling flow with $\sigma = 2, 4,$ and 6.

As the rotation increases, an increase in radial velocity values also takes place. Moreover, in the initial portion of the flow, where centrifugal forces dominate, $V_\xi > 0$, and in the degeneration portion of the rotation, $V_\xi < 0$.

The increase in the velocity of outflow to the wall in the initial portion of the flow results in an increase of j_ξ here (Fig. 1, curve 3), as was observed in the case of an increase in q_i. In this case, however, the displacement of the charge to the wall is chiefly due to convection processes, as a result of which the charged particles are carried into the vicinity of the grounded wall. As the rotation increases, this in turn leads to a decrease in the outflow current

$$I = 2\pi \int_0^1 j_x \xi \, d\xi,$$

the change of which is shown in Fig. 3, where curves 1-3 were plotted for $\mathrm{Re} = 160$, $q_i = 10$, and $\sigma = 0$, 6, and 9, respectively.

In addition to the integrated characteristic I, of interest as well is the density distribution of the longitudinal velocity component of current j_x over the channel radius. Figure 4 shows the radial distributions of j_x in the sections $x = 1$, 8, and 13 for $\sigma = 0$, 6, and 9 (curves 1-3, respectively). In the straight-through and slightly swirling streams, the flow of electric current takes place in a filament of radius $\xi \approx \xi_i$, a maximum of j_x is observed. In the region $\xi < \xi_i$, the dependence $j_x(\xi)$ is characterized by a dip, as in the case of the distribution of axial velocity. A further increase of rotation gives rise to a zone of electric back current in the region near the axis. At the same time, the charged particles execute motion in closed lines which differ slightly from the lines of flow at nonzero mobility of the charge or coincide with them when $b = 0$.

REFERENCES

1. A. B. Vatazhin, V. I. Grabovskii, V. A. Likhter, and V. I. Shul'gin, Electrogasdynamic Flows [in Russian], Nauka, Moscow (1983).
2. V. L. Granovskii, Electric Current in a Gas. Steady Current [in Russian], L. A. Sen and V. E. Golant (eds.), Nauka, Moscow (1971).
3. V. I. Grabovskii, "Electrohydrodynamic flow in a flat channel with an axial emitter electrode," Prikl. Mekh. Tekh. Fiz., No. 6, 35-40 (1970).
4. A. B. Vatazhin and V. I. Grabovskii, "Propagation of unipolarly charged jets in hydrodynamic flows," Prikl. Matem. Mekh., **37**, No. 1, 57-64 (1973).
5. A. D. Gosman, V. V. Pan, A. L. Ranchell, et al., Numerical Methods of Studying Viscous Flow [Russian translation], Mir, Moscow (1972).